Electricity & magnetism-1

Capacitor

Capacitor

A **capacitor** is a system of *two conductors* that carries *equal and opposite charges*. A capacitor *stores charge and energy* in the form of electro-static field.

We define **capacitance** as

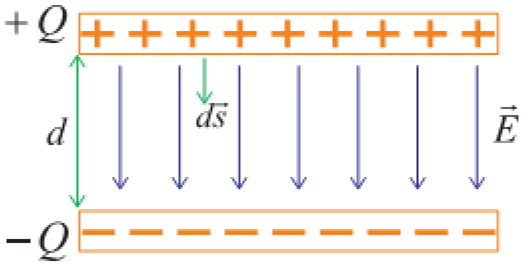
$$\boxed{C = \frac{Q}{V}} \qquad \text{Unit: Farad(F)}$$

where

$$\begin{aligned} Q &= \text{Charge on one plate} \\ V &= \text{Potential difference between the plates} \end{aligned}$$

Note: The C of a capacitor is a *constant* that depends only on its shape and material.
`i.e.` If we increase V for a capacitor, we can increase Q stored.

Parallel plate capacitor



Area of conducting

(1) Recall from Chapter 3 note,

$$|\vec{E}| = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

(2) Recall from Chapter 4 note,

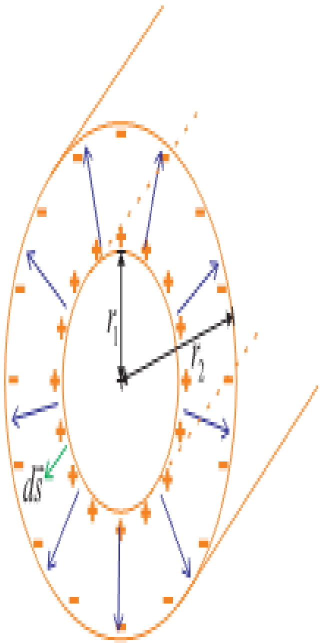
$$\Delta V = V_+ - V_- = -\int_-^+ \vec{E} \cdot d\vec{s}$$

Again, notice that this integral is independent of the path taken.
$\therefore$ We can take the path that is parallel to the $\vec{E}$-field.

$$\begin{aligned} \therefore \quad \Delta V &= \int_+^- \vec{E} \cdot d\vec{s} \\ &= \int_+^- E \cdot ds \\ &= \frac{Q}{\epsilon_0 A} \underbrace{\int_+^- ds}_{\text{Length of path taken}} \\ &= \frac{Q}{\epsilon_0 A} \cdot d \end{aligned}$$

(3) $\therefore$ $\boxed{C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}}$

Cylindrical capacitor



Consider two concentric cylindrical wire of innner and outer radii r_1 and r_2 respectively. The length of the capacitor is L where $r_1 < r_2 \ll L$.

(1) Using Gauss' Law, we determine that the E-field between the conductors is (`cf.` Chap3 note)

$$\vec{E} = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r}\,\hat{r} = \frac{1}{2\pi\epsilon_0} \cdot \frac{Q}{Lr}\,\hat{r}$$

where λ is charge per unit length

(2)

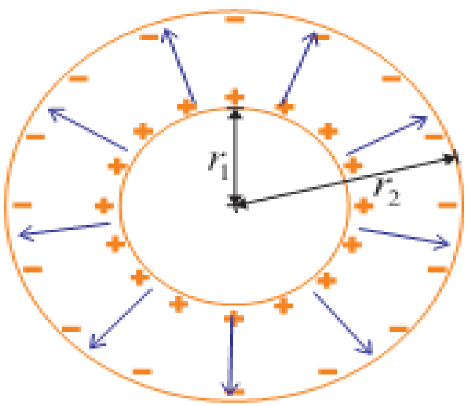
$$\Delta V = \int_{+}^{-} \vec{E} \cdot d\vec{s}$$

Again, we choose the path of integration so that $d\vec{s} \parallel \hat{r} \parallel \vec{E}$

$$\therefore \quad \Delta V = \int_{r_1}^{r_2} E\,dr = \frac{Q}{2\pi\epsilon_0 L} \underbrace{\int_{r_1}^{r_2} \frac{dr}{r}}_{\ln(\frac{r_2}{r_1})}$$

$$\therefore \quad \boxed{C = \frac{Q}{\Delta V} = 2\pi\epsilon_0 \frac{L}{\ln(r_2/r_1)}}$$

Spherical capacitor



$\vec{E}\,//\,\hat{r}$

Choose $d\vec{s}\,//\,\hat{r}$

For the space between the two conductors,

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}; \qquad r_1 < r < r_2$$

$$\Delta V = \int_+^- \vec{E}\cdot d\vec{s}$$

Choose $d\vec{s} \parallel \hat{r}$

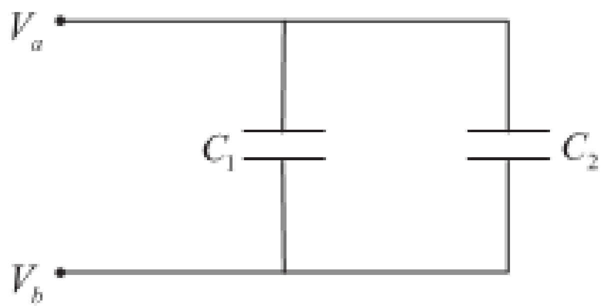
$$= \int_{r_1}^{r_2} \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}\,dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\boxed{C = 4\pi\epsilon_0\left[\frac{r_1 r_2}{r_2 - r_1}\right]}$$

Capacitors in combination

(a) Capacitors in Parallel



In this case, it's the *potential difference* $V = V_a - V_b$ that is the same across the capacitor.

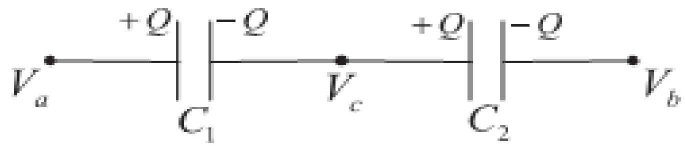
<u>BUT</u>: *Charge* on each capacitor different

$$\begin{aligned} \text{Total charge} \quad Q &= Q_1 + Q_2 \\ &= C_1 V + C_2 V \\ Q &= \underbrace{(C_1 + C_2)}_{\text{Equivalent capacitance}} V \end{aligned}$$

$\therefore$ For capacitors in parallel: $\boxed{C = C_1 + C_2}$

Continue….

(b) Capacitors in Series



The *charge across capacitors* are the same.

<u>BUT</u>: *Potential difference* (P.D.) across capacitors different

$$\Delta V_1 = V_a - V_c = \frac{Q}{C_1} \qquad \text{P.D. across } C_1$$
$$\Delta V_2 = V_c - V_b = \frac{Q}{C_2} \qquad \text{P.D. across } C_2$$

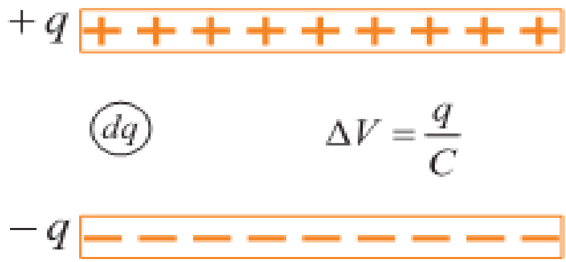
$\therefore$ Potential difference

$$\begin{aligned} \Delta V &= V_a - V_b \\ &= \Delta V_1 + \Delta V_2 \\ \Delta V &= Q\,(\frac{1}{C_1} + \frac{1}{C_2}) = \frac{Q}{C} \end{aligned}$$

where C is the **Equivalent Capacitance**

$$\therefore \quad \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Energy storage in capacitor



In charging a capacitor, *positive charge* is being moved from the *negative plate* to the *positive plate*.
$\Rightarrow$ NEEDS WORK DONE!

Suppose we move charge dq from $-ve$ to $+ve$ plate, *change in potential energy*

$$dU = \Delta V \cdot dq = \frac{q}{C}\, dq$$

Suppose we keep putting in a total charge Q to the capacitor, the *total potential energy*

$$U = \int dU = \int_0^Q \frac{q}{C}\, dq$$

$$\therefore \quad \boxed{U = \frac{Q^2}{2C} = \frac{1}{2} C \Delta V^2} \qquad (\because Q = C\Delta V)$$

The energy stored in the capacitor is stored in the **electric field** between the plates.

For energy density

$\therefore$ We can consider the E-field energy

$$\text{density u} = \frac{\text{Total energy stored}}{\text{Total volume with E-field}}$$

$$\therefore \quad u = \frac{U}{\underbrace{Ad}_{\text{Rectangular volume}}}$$

Recall

$$\begin{cases} C = \dfrac{\epsilon_0 A}{d} \\ E = \dfrac{\Delta V}{d} \quad \Rightarrow \quad \Delta V = Ed \end{cases}$$

$$\therefore \quad u = \frac{1}{2}\,\overbrace{(\frac{\epsilon_0 A}{d})}^{C} \cdot \overbrace{(\;Ed\;)^2}^{(\Delta V)^2} \cdot \overbrace{\frac{1}{Ad}}^{\frac{1}{Volume}}$$

Exercise

30-1 Ans. $q = C\Delta V = (50 \times 10^{-12}\,\mathrm{F})(0.15\,\mathrm{V}) = 7.5 \times 10^{-12}\,\mathrm{C}$

30-3 Ans. $q = C\Delta V = (26.0\times 10^{-6}\mathrm{F})(125\,\mathrm{V}) = 3.25\times 10^{-3}\mathrm{C}.$

30-5 Ans. $C = 2\pi\epsilon_0 \dfrac{L}{\ln(b/a)} = 2\pi(8.85\times 10^{-12}\,\mathrm{F/m})\dfrac{(0.0238\,\mathrm{m})}{\ln((9.15\mathrm{mm})/(0.81\mathrm{mm}))} = 5.46\times 10^{-13}\mathrm{F}$

Sample problem of ch# 30 and exercise questions 30-9 and 30-21.